

Joint Optimization of Spectrum Sensing and Accessing in Multiuser MISO Cognitive Networks

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Abstract—In this paper, a joint spectrum sensing and accessing optimization framework for a multiuser cognitive network is proposed to significantly improve spectrum efficiency. For such a cognitive network, there are two important and limited resources that should be distributed in a comprehensive manner, namely feedback bits and time duration. First, regarding the feedback bits, there are two components: sensing component (used to convey various users' sensing results) and accessing component (used to feedback channel state information). A large sensing component can support more users to perform cooperative sensing, which results in high sensing precision. However, a large accessing component is preferred as well, as it has a direct impact on the performance in the multiuser cognitive network when multi-antenna technique, such as zero-forcing beamforming (ZFBE), is utilized. Second, the tradeoff of sensing and accessing duration in a transmission interval needs to be determined, so that the sum transmission rate is optimized while satisfying the interference constraint. In addition, the above two resources are interrelated and inversive under some conditions. Specifically, sensing time can be saved by utilizing more sensing feedback bits for a given performance objective. Hence, the resources should be allocation in a jointly manner. Based on the joint optimization framework and the intrinsic relationship between the two resources, we propose two joint resource allocation schemes by maximizing the average sum transmission rate in a multiuser multi-antenna cognitive network. Simulation results show that, by adopting the joint resource allocation schemes, obvious performance gain can be obtained over the traditional fixed strategies.

Index Terms—MISO cognitive network, cooperative sensing, joint optimization, limited feedback, ZFBE.

I. INTRODUCTION

Within the past decade, wireless communication has undergone an unprecedentedly rapid growth to satisfy users' demands for various advanced services and applications. A stark reality resulted from the explosive development of wireless communication is that there is not enough available spectrum resource to ensure quality of services required by the users' applications. However, FCC's report [1] indicates that the radio spectrum is heavily under-utilized in time, frequency or space scales, which motivates a spectrum open policy that allows unlicensed users to opportunistically access the licensed spectrum when primary users are inactive. Inspired by that idea, a novel communication technology, namely cognitive radio, has been proposed and received considerable research attentions from both academe and industry [2]-[7].

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Since cognitive network is allowed to coexist with primary network by the spectrum administrator, it is important for the cognitive users to be transparent to the primary users by avoiding to create interference that will degrade the performance of primary network. In order to satisfy that rigorous requirement, cognitive networks are designed to include two crucial components: spectrum sensing [8]-[10] and dynamic spectrum accessing [11]-[13]. Spectrum sensing has two functions: 1) cognitive users should have strong enough ability to detect active primary users to guarantee normal communication of primary users, i.e. to increase the probability of detection; 2) cognitive users need to try their best to find the unoccupied spectrum so as to improve spectrum utilization efficiency, i.e. to decrease the probability of false alarm. So far, there have been several feasible spectrum sensing methods, for example, energy detection [14] [15], is a simple and popular method, which has a preferable performance without requiring any knowledge of primary signal a prior. Unfortunately, it has a low sensing precision when the ratio of sensing signal power to noise variance (sensing SNR) is low or noise variance is uncertain [16]. Although we can improve sensing accuracy by increasing sensing duration, it will reduce the duration for spectrum accessing for a given total duration constraint.

Recently, cognitive network equipped with multiple antennas is proved to have the ability of further enhancing the performance of both spectrum sensing [17] [18] and accessing [19] [20] without adding extra resource. By making use of its spatial dimensions, multiple copies of primary signal are obtained and fused at the detector, namely cooperative sensing [21]-[24]. If these copies are independent to each other, sensing time can be reduced accordingly, for a given sensing precision. As a result, there is more time available for accessing to improve transmission rate. Especially, in the multiuser multiantenna paradigm, there are several available performance-enhancing techniques for both spectrum sensing and accessing. On one hand, multiuser cooperative sensing can compensate for the insufficient capability of single user through combining multiple sensing information. On the other hand, multiuser MIMO technique is in favor of improving the transmission rate during spectrum accessing [25] [26]. It is worth pointing out that the performance of multiuser multiantenna cognitive network is greatly dependent on feedback resource [27], as both spectrum sensing and accessing require the related information feedback. Specifically, if there are more feedback amount available for spectrum sensing, more users are allowed to convey their sensing results to cognitive base station (BS), so that sensing precision can be improved. Meanwhile, if there are more feedback amount

available for spectrum sensing, users can convey more accurate channel state information (CSI) to cognitive BS. Cognitive BS performs to pre-process the signal to be transmitted in order to decrease interuser interference, and thus improve the performance. In order to optimize the performance, it is necessary to allocate the feedback resource between spectrum sensing and accessing for a given feedback resource constraint.

Common to most of the previous work on cognitive network is to separately study spectrum sensing and accessing. In fact, the two phases have a tight connection, especially for the multiuser scenario. In order to achieve the optimal performance, it is imperative to allocate the limited resource in the comprehensive sense, namely joint optimization of spectrum sensing and accessing. As discussed above, if there is a constraint on the total feedback amount, we should determine the proportion of feedback amount between sensing and accessing phases, so that the transmission rate is maximized. Similarly, for each transmission interval, we also need to determine the optimal interval of sensing and accessing duration. As a result, the problem of joint resource allocation for spectrum sensing and accessing is getting attention. As a pioneering work, [28] initiated the problem of time allocation between the two phases based on energy sensing to maximize the throughput of cognitive user. [29] extended the problem to the multiuser multichannel scenario. The previous works in the literatures only consider one dimensional resource allocation, for example the time dimension of spectrum sensing or spectrum accessing. In fact, the allocation of feedback amount is of equal importance to improve the performance of the multiuser MISO cognitive network together with time. Considering the correlation between the two resources, performance loss is inevitable if the allocation of feedback amount and time duration is optimized separately. In this paper, we address a joint spectrum sensing and accessing problem in a multiuser MISO cognitive network. By taking the maximization of the sum rate as the optimization objective, we construct a joint optimization framework of spectrum sensing and accessing, analyze the intrinsic relationship between feedback and time resources, and then derive two joint resource allocation schemes, which provide performance gain over the traditional fixed resource allocation schemes.

The rest of this paper is organized as follows. Section II gives a brief overview of the considered system model, and analyzes the adopted spectrum sensing and accessing strategies. Then, by maximizing the average sum transmission rate under some performance requirements, we derive two joint resource allocation schemes in Section III. Next, some simulation results are provided to verify the effectiveness of the proposed schemes in Section IV and the whole paper is concluded finally in Section V.

Notation: We use bold upper (lower) letters to denote matrices (column vectors), $(\cdot)^H$ to denote conjugate transpose, $(\cdot)'$ to denote the derivation, $E[\cdot]$ to denote expectation, $\|\cdot\|$ to denote the L_2 -norm of a vector, and $|\cdot|$ to denote the absolute value. The acronym i.i.d. means “independent and identically distributed”, pdf means “probability density function” and cdf means “cumulative distribution function”.

II. SYSTEM MODEL

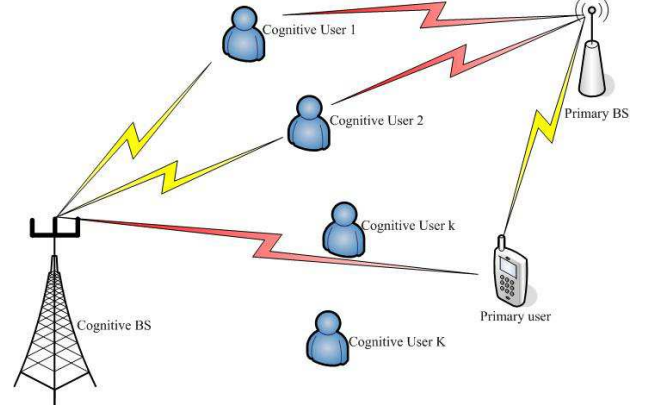


Fig. 1. The considered system block diagram.

In this paper, we consider a system including two networks, namely primary and cognitive networks, as seen in Fig.1. N_t antennas are mounted at the cognitive BS and K cognitive users equip with single antenna each. For ease of analysis, both primary BS and primary user are considered to have single antenna. It is assumed that the two networks are synchronized, and their transmissions are both in the form of time slot of length T . At the beginning of each time slot, multiple cognitive users cooperatively sense the state of the licensed spectrum in the duration of τ . If the spectrum is regarded as busy, then the cognitive network keeps silent in order to avoid the interference to primary network. Otherwise, the cognitive network transmits in the residual duration of $T - \tau$, so that the spectrum utilization efficiency is improved effectively.

Note that in an interweave cognitive network, the accessing opportunity of cognitive network is determined by the activity of primary network, but not the traffic characteristics, so we only preset the activity model [30]. First, the activity of primary network is assumed to be fully independent of cognitive network. Similar to [30], the channel occupancy by primary network is modeled as an “alternating renewal source” that alternates between busy and idle modes, where busy or idle denotes the channel is occupied or not by primary network, respectively. We use exponential distribution to describe the probability density function (pdf) of the busy and idle periods of each time slot, which can be expressed as

$$f_B(t) = \alpha e^{-\alpha t}, \quad (1)$$

and

$$f_I(t) = \beta e^{-\beta t}, \quad (2)$$

where α and β are the transition rates from busy to idle and from idle to busy, respectively. Then, the stationary probabilities for the spectrum to be busy and idle can be written as:

$$P_B = \frac{\beta}{\alpha + \beta}, \quad (3)$$

and

$$P_I = \frac{\alpha}{\alpha + \beta}, \quad (4)$$

respectively.

For an interweave cognitive network, it includes two important stages, namely spectrum sensing and accessing. In what follows, we introduce the spectrum sensing and accessing strategies adopted in this paper, respectively.

A. Spectrum Sensing

Due to the simplicity, we consider the use of energy detection [14] [15] for spectrum sensing. In order to further improve the spectrum efficiency and decrease the collision between the two networks, multiuser cooperative sensing is adopted in the MISO cognitive network.

By making use of the Nyquist sampling technique, for cognitive user k , the binary hypothesis test for spectrum sensing at time instant i takes the following form:

$$\begin{aligned}\mathcal{H}_0 &: y_k(i) = n_k(i) \\ \mathcal{H}_1 &: y_k(i) = g_k(i)s(i) + n_k(i),\end{aligned}\quad (5)$$

where $y_k(i)$ is the received signal at cognitive user k , $s(i)$ is the transmit signal from primary BS, $g_k(i)$ is the channel gain from primary BS to cognitive user k , and $n_k(i)$ is the zero mean additive white Gaussian noise, i.e., $n_k(i) \sim \mathcal{CN}(0, \sigma_k^2)$, which is independent of $s(i)$. Cognitive user k obtains the statistics test T_k by summing the $N = 2W\tau$ samplings, which is given by

$$T_k = \sum_{i=0}^{N-1} |y_k(i)|^2, \quad (6)$$

where W is the spectrum bandwidth. If the number of sample is large enough, according to the central limit theorem, the distribution of T_k can be approximated as

$$T_k \sim \begin{cases} \mathcal{N}(N\sigma_k^2, N\sigma_k^4), & \mathcal{H}_0 \\ \mathcal{N}(N(\sigma_k^2 + \sigma_s^2), N(\sigma_k^2 + \sigma_s^2)^2), & \mathcal{H}_1 \end{cases} \quad (7)$$

where σ_s^2 is the variance of the sensing signal $g_k(i)s(i)$. By employing energy detection, cognitive user k judges the spectrum state is 1 (denote busy) if T_k is greater than a threshold λ , or 0 (denote idle). Sequently, the sensing result is conveyed to cognitive BS by using 1 bit.

After receiving the feedback information from L cognitive users, the cognitive BS computes the final sensing result based on the “or” fusion criteria [31]. Specifically, only all L cognitive users consider the spectrum is idle, the spectrum state can be regarded as 0. Otherwise, the spectrum can not be utilized by cognitive network. Through such a cooperative sensing strategy, the detection probability $P(1|\mathcal{H}_1)$ and false-alarm probability $P(1|\mathcal{H}_0)$ can be expressed as

$$P(1|\mathcal{H}_1) = 1 - \prod_{l=1}^L P_l(0|\mathcal{H}_1),$$

and

$$P(1|\mathcal{H}_0) = 1 - \prod_{l=1}^L P_l(0|\mathcal{H}_0),$$

where $P_l(0|\mathcal{H}_1)$ and $P_l(0|\mathcal{H}_0)$ are the probabilities that cognitive user l judges the spectrum is idle when primary network is active and inactive, respectively. It is reasonably assumed that the cognitive users have the same $P_l(0|\mathcal{H}_1)$ and $P_l(0|\mathcal{H}_0)$

in the statistical sense due to their similar sense capabilities. Thereby, the detection and false-alarm probabilities can be rewritten as

$$\begin{aligned}P(1|\mathcal{H}_1) &= 1 - (1 - P_d)^L \\ &= 1 - \left(1 - Q\left(\frac{\lambda - N(\sigma_n^2 + \sigma_s^2)}{\sqrt{N(\sigma_s^2 + \sigma_n^2)^2}}\right)\right)^L,\end{aligned}\quad (8)$$

and

$$\begin{aligned}P(1|\mathcal{H}_0) &= 1 - (1 - P_f)^L \\ &= 1 - \left(1 - Q\left(\frac{\lambda - N\sigma_n^2}{\sqrt{N\sigma_n^4}}\right)\right)^L \\ &\approx LQ\left(\frac{\lambda - N\sigma_n^2}{\sqrt{N\sigma_n^4}}\right),\end{aligned}\quad (9)$$

where $P_d = Q\left(\frac{\lambda - N(\sigma_n^2 + \sigma_s^2)}{\sqrt{N(\sigma_s^2 + \sigma_n^2)^2}}\right)$ and $P_f = Q\left(\frac{\lambda - N\sigma_n^2}{\sqrt{N\sigma_n^4}}\right)$ are respectively the detection and false-alarm probabilities of an arbitrary cognitive user, which are derived based on the distribution of T_k in (7) and the assumption that all cognitive users have the same variance of the receive noise σ_n^2 , and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{y^2}{2})dy$ is the Q-function. (9) follows from the fact $(1 - x)^L \approx 1 - Lx$ when x is a sufficient small value. Based on (8) and (9), we have the following relationship

$$\begin{aligned}P(1|\mathcal{H}_0) &= LQ\left\{\frac{Q^{-1}\left(1 - (1 - P(1|\mathcal{H}_1))^{1/L}\right)}{\sqrt{N\sigma_n^4}}\right. \\ &\quad \times \left.\frac{\sqrt{N(\sigma_s^2 + \sigma_n^2)^2 + N\sigma_s^2}}{\sqrt{N\sigma_n^4}}\right\} \\ &= LQ\left\{Q^{-1}\left(1 - (1 - P(1|\mathcal{H}_1))^{1/L}\right)\right. \\ &\quad \times \left.(1 + \xi) + \sqrt{2W\tau\xi}\right\},\end{aligned}\quad (10)$$

where $\xi = \frac{\sigma_s^2}{\sigma_n^2}$ is the so-called sensing SNR of the received sensing signal.

B. Spectrum Accessing

If primary network is judged as inactive based on the fused sensing information, cognitive network accesses the spectrum opportunistically. For a multiuser network, it is proved that the system performance is improved with the increase of feedback amount about channel state information. In this paper, we adopt limited feedback zero-forcing beamforming (ZFBBF) as the spectrum accessing strategy. For convenience of analysis, we assume that all cognitive users always have information to receive and they are scheduled based on round robin policy. Specifically, cognitive users are serviced by an predetermined order independent of channel state. During each time slot, a fixed number of cognitive users, such as N_t , are coordinated to access the available spectrum based on a certain user scheduling scheme, e.g. round bin.

In this paper, we adopt codebook based limited feedback scheme. When the spectrum is open, cognitive user k , who is scheduled in current slot, selects an optimal codeword $\mathbf{h}_{k,\text{opt}}$

from the predetermined codebook \mathcal{H}_k of size 2^B based on the instantaneous CSI \mathbf{h}_k , where $\mathcal{H}_k = \{\hat{\mathbf{h}}_{k,1}, \hat{\mathbf{h}}_{k,2}, \dots, \hat{\mathbf{h}}_{k,2^B}\}$. The codeword selection criteria can be expressed as

$$\hat{\mathbf{h}}_{k,\text{opt}} = \arg \max_{1 \leq j \leq 2^B} |\tilde{\mathbf{h}}_k^H \hat{\mathbf{h}}_{k,j}|^2, \quad (11)$$

where $\tilde{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ is the direction vector of \mathbf{h}_k . The other scheduled users select their optimal codewords from different quantization codebooks and convey the corresponding selected codeword indexes to cognitive BS. Assuming that the channels are i.i.d. and block fading. In other words, the channel keeps constant during a time slot and fades independently slot by slot. It is worth pointing out that the duration of CSI feedback is quite small and is negligible, we do not consider it in this paper. After receiving the feedback information of N_t current scheduled cognitive users, cognitive BS determines the optimal transmit beams $\mathbf{w}_k, k = 1, \dots, N_t$ by making use of ZFBF design method. Specifically, for the k th user, we first construct its complementary channel matrix

$$\tilde{\mathbf{H}}_k = [\hat{\mathbf{h}}_{1,\text{opt}}, \dots, \hat{\mathbf{h}}_{k-1,\text{opt}}, \hat{\mathbf{h}}_{k+1,\text{opt}}, \dots, \hat{\mathbf{h}}_{N_t,\text{opt}}],$$

where $\hat{\mathbf{h}}_{k-1,\text{opt}}$ is the optimal codeword selected by the $(k-1)$ th user based on (11). Taking singular value decomposition (SVD) to $\tilde{\mathbf{H}}_k$, if \mathbf{V}_k^\perp is the matrix composed of the right singular vectors with respect to zero singular values, then \mathbf{w}_k is a normalized vector spanned by the space of \mathbf{V}_k^\perp , so that we have

$$\hat{\mathbf{h}}_{u,\text{opt}}^H \mathbf{w}_k = 0, \quad k \leq u \leq N_t, u \neq k$$

It is assumed that x_k is the expected normalized signal of the k th current scheduled user, then its received signal can be expressed as

$$y_k = \sqrt{\frac{P}{N_t}} \sum_{u=1}^{N_t} \mathbf{h}_k^H \mathbf{w}_u x_u + n_k + n_s, \quad (12)$$

where P is the total transmit power of cognitive BS, which is equally allocated to N_t users. n_k is the additive Gaussian white noise with zero mean and covariance σ_n^2 for all users. Due to miss-detection, we consider the interference n_s from primary network. For ease of analysis, it is assumed that n_s is an i.i.d. complex Gaussian random variable with zero mean and covariance σ_s^2 . Hence, the ratio of the received signal to interference and noise (SINR) for the k th user can be expressed as

$$\begin{aligned} \rho_k &= \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{N_t(\sigma_s^2 + \sigma_n^2)/P + \sum_{u=1, u \neq k}^{N_t} |\mathbf{h}_k^H \mathbf{w}_u|^2} \\ &= \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{1/\gamma + \sum_{u=1, u \neq k}^{N_t} |\mathbf{h}_k^H \mathbf{w}_u|^2}, \end{aligned} \quad (13)$$

where $\gamma = \frac{P}{N_t(\sigma_n^2 + \sigma_s^2)} = \frac{\varrho}{N_t(1+\xi)}$, $\varrho = \frac{P}{\sigma_n^2}$ is the average transmit SNR at cognitive BS, and $\xi = \frac{\sigma_s^2}{\sigma_n^2}$ is the sensing SNR.

As a result, the average sum transmission rate of the multiuser MISO cognitive network based on limited feedback ZFBF in the presence of mis-detection can be expressed as

$$\begin{aligned} R &= E \left[\sum_{k=1}^{N_t} \log_2(1 + \rho_k) \right] \\ &= N_t E [\log_2(1 + \rho_k)], \end{aligned} \quad (14)$$

where (14) holds true because all the channels are i.i.d. Clearly, in order to compute the average sum transmission rate, the key is the achievement of the pdf of the received SINR ρ_k . For the pdf and cdf of ρ_k , we have the following lemma:

Lemma 1: For limited feedback zero-forcing beamforming in the setting of N_t BS antennas, N_t single antenna users and the quantization codebooks of size 2^B , the pdf and cdf of SINR are $f_{\rho_k}(x) = 1/\gamma \exp(-x/\gamma)(1 + \delta x)^{-(N_t-1)} + \delta(N_t - 1) \exp(-x/\gamma)(1 + \delta x)^{-N_t}$ and $F_{\rho_k}(x) = 1 - \frac{\exp(-x/\gamma)}{(1 + \delta x)^{N_t-1}}$ respectively, where $\delta = 2^{-\frac{B}{N_t-1}}$.

Proof: Please refer to Appendix. ■

Based on the above pdf and cdf of the received SINR, the average sum transmission rate can be computed as

$$\begin{aligned} R &= N_t E [\log_2(1 + \rho_k)] \\ &\approx N_t \log_2(e) E [\ln(1 + \rho_k)] \\ &= N_t \log_2(e) \int_0^\infty \ln(1 + x) f_{\rho_k}(x) dx \\ &= -N_t \log_2(e) \int_0^\infty \ln(1 + x) (1 - F_{\rho_k}(x))' dx \\ &= N_t \log_2(e) \int_0^\infty \frac{1 - F_{\rho_k}(x)}{1 + x} dx \\ &= \frac{N_t \log_2(e)}{\delta^{N_t-1}} \int_0^\infty \frac{\exp(-x/\gamma)}{(x + 1)(x + \delta^{-1})^{N_t-1}} dx \\ &= \frac{N_t \log_2(e)}{\delta^{N_t-1}} I_1(\gamma^{-1}, \delta^{-1}, N_t - 1) \end{aligned} \quad (15)$$

$$= 2^B N_t \log_2(e) I_1\left(\gamma^{-1}, 2^{\frac{B}{N_t-1}}, N_t - 1\right), \quad (16)$$

where

$$\begin{aligned} I_1(a, b, M) &= \int_0^\infty \frac{\exp(-ax)}{(x + 1)(x + b)^M} dx \\ &= \sum_{i=1}^M (-1)^{i-1} (1 - b)^{-i} I_2(a, b, M - i + 1) \\ &\quad + (b - 1)^{-M} I_2(a, 1, 1), \end{aligned} \quad (17)$$

$$I_2(a, b, M) = \begin{cases} \exp(ab) E_1(ab) & M = 1 \\ \sum_{i=1}^{M-1} \frac{(i-1)!}{(M-1)!} \frac{(-a)^{M-i-1}}{b^i} \\ \quad + \frac{(-a)^{M-1}}{(M-1)!} \exp(ab) E_1(ab), & M \geq 2 \end{cases} \quad (18)$$

and $E_1(x)$ is the exponential-integral function of the first order. Hence, we derive the average sum transmission rate in presence of mis-detection as a function of codebook size B .

III. JOINT OPTIMIZATION OF SPECTRUM SENSING AND ACCESSING

This section concentrates on the parameter optimization of spectrum sensing and accessing, so as to maximize the average sum transmission rate. As analyzed above, spectrum sensing and accessing have a tight connection and together they determine the system performance. In order to maximize the average transmission rate, it is imperative to jointly optimize spectrum sensing and accessing through feedback amount and time duration allocation.

A. The Fixed Number of Accessing Users

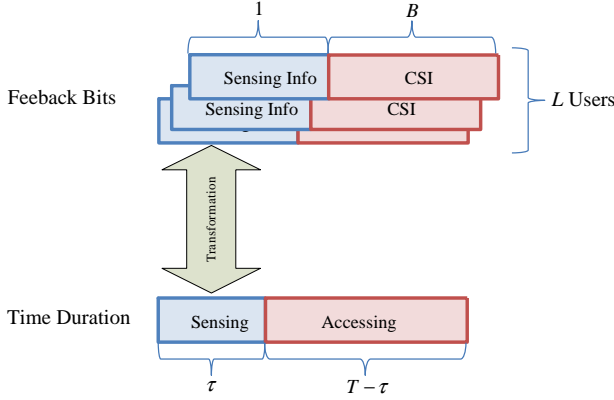


Fig. 2. The block diagram of joint resource allocation.

For multiuser cooperative spectrum sensing, if one cognitive user uses 1 bit to inform cognitive BS its sensing result, when there are L users involved in, the total feedback bits for spectrum sensing will be L bits for each time slot. For spectrum accessing based on ZFBF, one scheduled user uses B bits to quantize its instantaneous CSI. Thereby, the total feedback amount for spectrum sensing and accessing is $L + N_t B \leq \iota$, where ι is the constraint on feedback amount. In addition, for time resource, we assume the duration of sensing time is τ and the left time $T - \tau$ is allocated for accessing, as seen in Fig.2. In this paper, we maximize the average sum transmission rate while satisfying the protection to primary network as the optimization objective, which can be formulated as the following problem:

$$J_1 = \max_{\tau, L, B} \left(1 - \frac{\tau}{T}\right) \left(P_I (1 - P(1|\mathcal{H}_0)) R_1 + P_B (1 - P(1|\mathcal{H}_1)) R_2 \right)$$

Subject to

$$\begin{cases} P(1|\mathcal{H}_0) \leq P_0 & (19a) \\ P(1|\mathcal{H}_1) \geq P_1 & (19b) \\ 0 \leq \tau \leq T & (19c) \\ L + N_t B \leq \iota, & (19d) \end{cases}$$

where P_0 and P_1 are respectively the upper bound of false-alarm probability and the lower bound of detection probability preset to improve spectrum efficiency and

protect the normal communication of primary network. $R_1 = 2^B N_t \log_2(e) I_1 \left(N_t \sigma_n^2 / P, 2^{\frac{B}{N_t-1}}, N_t - 1 \right)$ and $R_2 = 2^B N_t \log_2(e) I_1 \left(N_t (\sigma_n^2 + \sigma_s^2) / P, 2^{\frac{B}{N_t-1}}, N_t - 1 \right)$ are the average sum transmission rates of cognitive network when primary network is idle and busy, respectively, according to (16). Examining the objective function, we find that it is a decreasing function of detection function $P(1|\mathcal{H}_1)$, and considering it is lower bounded by P_1 , so the objective function is maximized when $P(1|\mathcal{H}_1) = P_1$. Then, combining (10) and (19a), we have

$$LQ \left(Q^{-1} \left(1 - (1 - P(1|\mathcal{H}_1))^{1/L} \right) (1 + \xi) + \sqrt{2W\tau\xi} \right) \leq P_0.$$

Solving the above inequality, we get

$$\tau \geq \frac{\left(Q^{-1} (P_0/L) - Q^{-1} \left(1 - (1 - P_1)^{1/L} \right) (1 + \xi) \right)^2}{2W\xi^2}. \quad (20)$$

Thus, the optimization problem is reduced as

$$J_2 = \max_{\tau, L, B} \left(1 - \frac{\tau}{T}\right) \left(P_I (1 - P(1|\mathcal{H}_0)) R_1 + P_B (1 - P_1) R_2 \right)$$

Subject to

$$\begin{cases} \left(Q^{-1} (P_0/L) - Q^{-1} \left(1 - (1 - P_1)^{1/L} \right) (1 + \xi) \right)^2 \\ \leq \tau \leq T \\ L + N_t B \leq \iota. \end{cases} \quad \begin{matrix} (21a) \\ (21b) \end{matrix}$$

Unfortunately, the above optimization problem is a mixed integer programming problem, it is difficult to obtain a close-form expression of optimal τ , L and B . Intuitively, the optimal solution can be obtained by exhaustive search. Specifically, for a given feedback combination, we can get the optimal sensing duration by solving a convex optimization related to τ . Then, through comparing the average sum rate of all feedback combinations, we can get the optimal one. However, if the feedback amount constraint ι is large, the computation complexity is unbearable. Alternatively, we attempt to seek a suboptimal method to jointly decide the three parameters of spectrum sensing and accessing, so that we can achieve a feasible solution for practical implementation.

First, examining the objective function, it has the following appealing property:

Lemma 2: Given L and B , J_2 is a concave function with respect to τ .

Proof: Replacing $P(1|\mathcal{H}_1)$ with $LQ \left(Q^{-1} \left(1 - (1 - P_1)^{1/L} \right) (1 + \xi) + \sqrt{2W\tau\xi} \right)$ according to (10), the objective function can be rewritten as $V(\tau) = \left(1 - \frac{\tau}{T}\right) \left((1 - LQ(Q^{-1}(1 - (1 - P_1)^{1/L})(1 + \xi) + \sqrt{2W\tau\xi}))A + C \right)$, where $A = P_I R_1$ and $C = P_B (1 - P_1) R_2$ are two positive constants independent

of τ . Taking two-order derivation to $V(\tau)$ with respect to τ , we have

$$\begin{aligned} \frac{d^2 V(\tau)}{d\tau^2} &= -\left(1 - \frac{\tau}{T}\right) \left\{ \frac{L}{2\pi} \exp\left(-\frac{(U + D\sqrt{\tau})^2}{2}\right) \right. \\ &\quad \times \left(\frac{D^2}{2} + \frac{UD}{4} \tau^{-1/2} + \frac{\tau^{-1}}{2} \right) \frac{D}{2} \tau^{-1/2} \Big\} \\ &\quad - \frac{2}{T} \frac{L}{2\pi} \exp\left(-\frac{(U + D\sqrt{\tau})^2}{2}\right) \frac{D}{2} \tau^{-1/2} \\ &< 0, \end{aligned} \quad (22)$$

where $U = Q^{-1}(1 - (1 - P_1)^{1/L})(1 + \xi)$ and $D = \sqrt{2W}\xi$. Hereby, J_2 is a concave function with respect to τ . ■

Since the constraint (21a) is linear, when given L and B , we can derive the optimal sensing time τ by Lagrange multiplier method. However, given τ , solving L and B is an integer programming problem. Intuitively, J_2 is an increasing function of L and B because they are beneficial to improve the system performance, so the total feedback amount ι should be utilized as completely as possible. To solve such an integer programming problem, greedy algorithm is a simple and powerful choice. Specifically, from a given initial values, at each step, L or B is added by 1 to compare the performance gain. If the performance gain caused by 1 increment on L is larger, then $L = L + 1$. Otherwise, $B = B + 1$. Therefore, with the purpose of joint optimization of τ , L and B , we can first allocate the feedback bits by greedy algorithm for a given sensing time, and then update the optimal sensing time based on the predetermined L and B . The iteration stops until all feedback bits are used. Thus, the whole procedure can be summarized as below.

Algorithm 1

- 1) Initialization: given α , β , W , P_0 , P_1 , T , ϱ , ξ and ι , and set $L = 1$, $B = 1$, $\tau_1(L, B) = \frac{(Q^{-1}(P_0/L) - Q^{-1}(1 - (1 - P_1)^{1/L})(1 + \xi))^2}{2W\xi^2}$, $\tau_u = T$ and $V(\tau, L, B) = (1 - \frac{\tau}{T}) \left((1 - LQ(Q^{-1}(1 - (1 - P_1)^{1/L})(1 + \xi) + \sqrt{2W}\tau\xi))P_I R_1 + P_B(1 - P_1)R_2 \right)$.
- 2) Let $L_0 = L + 1$, $B_0 = B + 1$, $\tau_{0,0} = \tau_1(L_0, B)$, $\tau_{0,1} = \tau_u$, $\tau_{1,0} = \tau_1(L, B_0)$ and $\tau_{1,1} = \tau_u$.
- 3) Compute $V(\tau_{0,0}, L_0, B)$ and $V(\tau_{0,1}, L_0, B)$. If $V(\tau_{0,0}, L_0, B) \leq V(\tau_{0,1}, L_0, B)$, then $\tau_{0,0} = \frac{\tau_{0,0} + \tau_{0,1}}{2}$. Otherwise, $\tau_{0,1} = \frac{\tau_{0,0} + \tau_{0,1}}{2}$. If $\tau_{0,1} - \tau_{0,0} > \varepsilon$ (ε is a quite small real value), then repeat from 3), otherwise let $\tau_0 = \frac{\tau_{0,0} + \tau_{0,1}}{2}$.
- 4) Compute $V(\tau_{1,0}, L, B_0)$ and $V(\tau_{1,1}, L, B_0)$. If $V(\tau_{1,0}, L, B_0) \leq V(\tau_{1,1}, L, B_0)$, then $\tau_{1,0} = \frac{\tau_{1,0} + \tau_{1,1}}{2}$. Otherwise, $\tau_{1,1} = \frac{\tau_{1,0} + \tau_{1,1}}{2}$. If $\tau_{1,1} - \tau_{1,0} > \varepsilon$ (ε is a quite small real value), then repeat from 4), otherwise let $\tau_1 = \frac{\tau_{1,0} + \tau_{1,1}}{2}$.
- 5) If $V(\tau_0, L_0, B) \geq V(\tau_1, L, B_0)$, then $L = L_0$ and $\tau = \tau_0$. Otherwise, $B = B_0$ and $\tau = \tau_1$.
- 6) If $(L + 1) + N_t B \leq \iota$ and $L + N_t(B + 1) \leq \iota$, then repeat from 2). If $(L + 1) + N_t B \leq \iota$ and $L + N_t(B + 1) > \iota$,

then $L = L_0 = \iota - N_t B$, compute τ_0 according to 3), let $\tau = \tau_0$.

Remark: During the above joint optimization, we compute the number of sensing users L and codebook size B by the greedy algorithm. Note that the sensing duration is obtained by the bisection method, so the computation amount of Algorithm 1 is ι times as many as that of the bisection method at most. Although it is not optimal, it achieves a preferable tradeoff between system performance and implementation complexity to some extent compared with the exhaustion method.

B. The Variable Number of Accessing Users

It is worth noting that in the above, we fix the number of accessing users as N_t . In fact, N_t just is the upper bound of the number of accessing users K_0 , which is a variable scaling from 1 to N_t depending on the network condition. Generally speaking, if the network is noise and interference (from primary network) limited, a large number of accessing users is better to improve the performance. Otherwise, if the network is interuser interference limited, a small number of accessing users is preferable. The number of accessing users has a direct impact on the allocation of feedback bits, and thus time duration. In addition, the constraints on detection and false-alarm probabilities would also affect the number of accessing users. Hence, as a parameter of joint optimization of spectrum sensing and accessing, it is imperative to determine the optimal number of accessing user K_0 according to network conditions and sensing constraints. If we assume the number of accessing users is K_0 , then the received SINR for the k th user with miss-detection can be expressed as

$$\rho_k = \gamma |\mathbf{h}_k^H \mathbf{w}_k|^2, \quad (23)$$

for $K_0 = 1$, where $\mathbf{w}_k = \hat{\mathbf{h}}_k$ is the maximum ratio transmission (MRT). MRT is selected because it can maximize the average transmission rate with limited feedback when no inter-user interference cancelation is needed, and

$$\begin{aligned} \rho_k &= \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{M(\sigma_s^2 + \sigma_n^2)/P + \sum_{u=1, u \neq k}^{K_0} |\mathbf{h}_k^H \mathbf{w}_u|^2} \\ &= \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{1/\gamma + \sum_{u=1, u \neq k}^{K_0} |\mathbf{h}_k^H \mathbf{w}_u|^2}, \end{aligned} \quad (24)$$

for $2 \leq K_0 \leq N_t$.

For the case of $2 \leq K_0 \leq N_t$, based on the similar analysis in Appendix, we could get the corresponding pdf and cdf of received SINR in the case of K_0 accessing users. Averaging the instantaneous sum transmission rate over the pdf of received SINR, we have

$$R = 2^{\frac{B(K_0-1)}{N_t-1}} K_0 \log_2(e) I_1 \left(\gamma^{-1}, 2^{\frac{B}{N_t-1}}, K_0 - 1 \right). \quad (25)$$

Following [32], the average transmission rate of MRT is given

by

$$R = \log_2(e) \left\{ \exp(\gamma^{-1}) \sum_{k=0}^{N_t-1} E_{k+1}(\gamma^{-1}) - \int_0^1 (1 - (1-v)^{N_t-1})^{2^B} \frac{N_t}{v} \exp((\gamma v)^{-1}) \times E_{N_t+1}((\gamma v)^{-1}) dv \right\}, \quad (26)$$

where $E_n(x) = \int_1^\infty \exp(-xt)x^{-n}dt$ is the n th exponential integral. In order to encourage the users to sense the spectrum, we stipulate that only the sensing users are allowed to access the available spectrum. In other words, the number of sensing users L is equal to the number of accessing users K_0 . Hence, the joint optimization of spectrum sensing and accessing is equivalent to the following optimization problem

$$J_3 = \max_{\tau, L, B, K_0} \left(1 - \frac{\tau}{T} \right) \left(P_I (1 - P(1|\mathcal{H}_0)) R_1 + P_B (1 - P_1) R_2 \right)$$

Subject to

$$\begin{cases} \frac{(Q^{-1}(P_0/L) - Q^{-1}(1 - (1 - P_1)^{1/L})(1 + \xi))^2}{2W\xi^2} \\ \leq \tau \leq T \\ L + LB \leq \iota, \end{cases} \quad (27a)$$

$$(27b)$$

where $R_1 = 2^{\frac{B(L-1)}{N_t-1}} L \log_2(e) I_1(L\sigma_n^2/P, 2^{\frac{B}{N_t-1}}, L - 1)$ and $R_2 = 2^{\frac{B(L-1)}{N_t-1}} L \log_2(e) I_1(L(\sigma_n^2 + \sigma_s^2)/P, 2^{\frac{B}{N_t-1}}, L - 1)$ for $2 \leq K_0 = L \leq N_t$, and $R_1 = \log_2(e) (\exp(\sigma_n^2/P) \sum_{k=0}^{N_t-1} E_{k+1}(\sigma_n^2/P) - \int_0^1 (1 - (1-v)^{N_t-1})^{2^B} \frac{N_t}{v} \exp(\sigma_n^2/(Pv)) E_{N_t+1}(\sigma_n^2/(Pv)) dv)$ and $R_2 = \log_2(e) (\exp((\sigma_n^2 + \sigma_s^2)/P) \sum_{k=0}^{N_t-1} E_{k+1}(\gamma^{-1}(\sigma_n^2 + \sigma_s^2)/P) - \int_0^1 (1 - (1-v)^{N_t-1})^{2^B} \frac{N_t}{v} \exp((\sigma_n^2 + \sigma_s^2)/(Pv)) E_{N_t+1}((\sigma_n^2 + \sigma_s^2)/(Pv)) dv)$ for $K_0 = L = 1$. Similar to J_2 , this problem is also a mixed integer programming problem, so that it is difficult to give a close-form expression of the optimal solution. Based on the same idea of algorithm 1, the number of accessing users, the number of sensing users, codebook size and sensing time can be jointly determined through the following algorithm

Algorithm 2

- 1) Initialization: given $\alpha, \beta, W, P_0, P_1, T, \varrho, \xi$ and ι , and set $L = 1, B = 1, \tau(L, B) = \frac{(Q^{-1}(P_0/L) - Q^{-1}(1 - (1 - P_1)^{1/L})(1 + \xi))^2}{2W\xi^2}$, $\tau_u = T$ and $V(\tau, L, B) = (1 - \frac{\tau}{T}) \left((1 - LQ(Q^{-1}(1 - (1 - P_1)^{1/L})(1 + \xi) + \sqrt{2W\tau\xi})) P_I R_1 + P_B (1 - P_1) R_2 \right)$.

- 2) Let $L_0 = L + 1, B_0 = B + 1, \tau_{0,0} = \tau(L_0, B), \tau_{0,1} = \tau_u, \tau_{1,0} = \tau(L, B_0)$ and $\tau_{1,1} = \tau_u$.
- 3) Compute $V(\tau_{0,0}, L_0, B)$ and $V(\tau_{0,1}, L_0, B)$. If $V(\tau_{0,0}, L_0, B) \leq V(\tau_{0,1}, L_0, B)$, then $\tau_{0,0} = \frac{\tau_{0,0} + \tau_{0,1}}{2}$. Otherwise, $\tau_{0,1} = \frac{\tau_{0,0} + \tau_{0,1}}{2}$. If $\tau_{0,1} - \tau_{0,0} > \varepsilon$ (ε is a quite small real value), then repeat from 3), otherwise let $\tau_0 = \frac{\tau_{0,0} + \tau_{0,1}}{2}$.
- 4) Compute $V(\tau_{1,0}, L, B_0)$ and $V(\tau_{1,1}, L, B_0)$. If $V(\tau_{1,0}, L, B_0) \leq V(\tau_{1,1}, L, B_0)$, then $\tau_{1,0} = \frac{\tau_{1,0} + \tau_{1,1}}{2}$. Otherwise, $\tau_{1,1} = \frac{\tau_{1,0} + \tau_{1,1}}{2}$. If $\tau_{1,1} - \tau_{1,0} > \varepsilon$ (ε is a quite small real value), then repeat from 4), otherwise let $\tau_1 = \frac{\tau_{1,0} + \tau_{1,1}}{2}$.
- 5) If $V(\tau_0, L_0, B) \geq V(\tau_1, L, B_0)$, then $L = L_0$ and $\tau = \tau_0$. Otherwise, $B = B_0$ and $\tau = \tau_1$.
- 6) If $L + 1 + (L + 1)B \leq \iota, L + L(B + 1) \leq \iota$ and $L + 1 \leq N_t$, then repeat from 2). If $L + 1 + (L + 1)B \leq \iota, L + L(B + 1) > \iota$ and $L + 1 \leq N_t$, then $L = L_0 = \lfloor \frac{\iota}{B+1} \rfloor$, compute τ_0 according to 3), let $\tau = \tau_0$. If $L + 1 + (L + 1)B > \iota$ and $L + L(B + 1) \leq \iota$, then $B = B_0 = \lfloor \frac{\iota}{L} \rfloor - 1$, compute τ_1 according to 4), let $\tau = \tau_1$.
- 7) Set $K_0 = L$.

Notice that it is unnecessary to confine that the number of accessing users to be equal to the number of sensing users in the joint optimization. However, if the number of sensing users is independent, there will be one more integer optimization variable, and thus the complexity of joint optimization will be increased while the performance gain is limited.

In this section, we have derived the joint optimization schemes of spectrum sensing and accessing, namely joint resource allocation algorithms by maximizing the average sum transmission rate. In fact, according to the aforementioned relationship of time and feedback resources, we can realize the tradeoff between them for a given performance requirement. For example, we can reduce the total feedback amount by increasing sensing time, which is appealing to feedback limited systems. Additionally, it is worth pointing out that although the above schemes are derived based on the exponential activity model of primary network, they are applicable for an arbitrary activity model. As analyzed above, as long as the stationary probabilities for the spectrum to be busy P_B and idle P_I are given, the corresponding joint resource allocation algorithm can be obtained.

IV. SIMULATION RESULTS AND PERFORMANCE ANALYSIS

In order to verify the validity of our theoretical claims, we present several simulations in different scenarios. The simulation parameters are set according to Tab.I. In order to show the advantages of the proposed joint optimization schemes explicitly, we compare them with the traditional fixed resource allocation schemes, which have fixed τ, B, L and K_0 when given T and ι . Hereafter, we use Algorithm 1, Algorithm 2 and Traditional Algorithm to denote the proposed algorithm 1, algorithm 2 and the fixed resource allocation algorithm, respectively. Note that the simulation results are obtained by averaging the sum rate over 10000 channel samples.

TABLE I
SIMULATION PARAMETER TABLE

Parameter	Description	Value
N_t	Antenna Number	4
T	Length of Time Slot	10ms
W	Spectrum Bandwidth	5KHz
α	Transition Rate from Busy to Idle	0.9
β	Transition Rate from Idle to Busy	0.1
P_1	The Lower Bound on Detection Probability	0.9
P_0	The Upper Bound on False-Alarm Probability	0.1
ι	Feedback Bits	10, 20, 30, 40
ξ	Sensing SNR	-6dB, -4dB, -2dB, 0dB
B	Codebook Size	Optimization Variable
L	Number of Accessing User	Optimization Variable
τ	Sensing Duration	Optimization Variable

First, we investigate the impact of feedback constraint on the joint optimization of spectrum sensing and accessing when given $\xi = 0\text{dB}$ and $\varrho = 15\text{dB}$. Tab.II shows the resource allocation results with different feedback constraints. Note that traditional algorithm allocates time or feedback source separately as long as the requirements of detection and false-alarm probabilities are met. It is found that, for the two proposed joint optimization algorithms, when there is a strict feedback constraint, i.e. small ι , more time is allocated for spectrum sensing to increase the precision of cooperative sensing. With the increase of ι , sensing time τ decreases accordingly while the number of sensing users L increases. This is because increasing L is more beneficial to improve sensing precision than increasing τ . Meanwhile, more time can be allocated to improve the average sum rate. In addition, more feedback bits are distributed to CSI conveyance, namely enlarging codebook size. There are two reasons: first, when satisfying sensing precision, a large B can decrease interuser interference, and thus improve average sum rate; second, limited by the probability of spectrum idle, namely P_I , further increase in L hardly adds the probability of spectrum accessing. As a simple example, for algorithm 1, although L is not limited by N_t , it is still a relatively small value when ι is large. Fig.3 presents the corresponding average sum rates of the above resource allocation results. It can be seen that the average sum rate of traditional algorithm nearly keeps constant when ι is greater than 20, since the added feedback bits are used to increase L . However, large L is useless as discussed above. The proposed joint optimization algorithms perform better than the traditional scheme, and the performance gain becomes larger with the increase of ι . Algorithm 2 has an evident advantage over algorithm 1 under the conditions of small ι , because Algorithm 2 admits a small number of accessing users, so that the interuser interference is decreased and codebook size B can be enlarged. With the increase of ι , the performance gap between algorithm 2 and algorithm 1 reduces gradually until they have the same performance, due to the same allocation results.

Second, we study the role of sensing SNR ξ in joint optimization of spectrum sensing and accessing when given $\iota = 15$ and $\varrho = 15\text{dB}$. As we know, sensing SNR has two contrary effects on joint optimization and thus average sum rate. On one hand, high ξ can reduce the resource consumption

TABLE II
JOINT OPTIMIZATION WITH DIFFERENT FEEDBACK CONSTRAINTS

	ι	10	20	30	40
Traditional Algorithm	B	2	4	4	4
	L	2	4	12	24
	τ	2.0	2.0	2.0	2.0
Algorithm 1	B	2	4	7	9
	L	2	4	2	4
	τ	0.8452	0.4467	0.8361	0.4467
Algorithm 2	B	4	9	9	9
	L	2	2	3	4
	τ	0.8361	0.8361	0.5789	0.4467

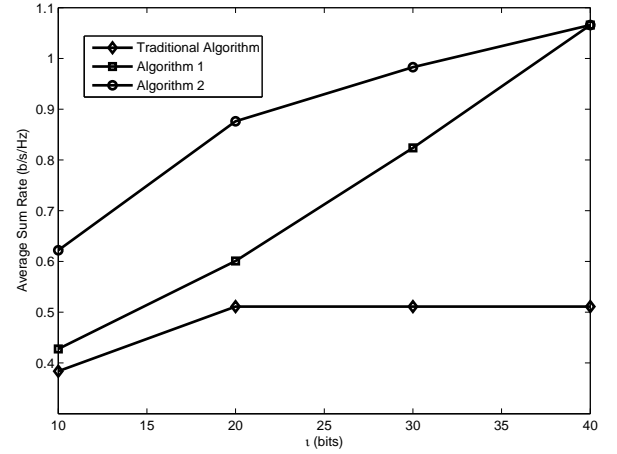


Fig. 3. Comparison of average sum rate with different feedback constraints.

for spectrum sensing, hence more time and feedback can be used for accessing. On the other hand, high ξ means strong interference from primary network and results in the descent of average sum rate. Herein, we consider the scenario with low sensing SNR. As seen in Tab.III, when sensing SNR is quite low, such as -6dB , feedback bits are allocated to spectrum sensing as much as possible in order to enhance the sensing precision through cooperation. For example, algorithm 2 uses the upper bound of the feedback bits for sensing. Once sensing SNR increases, more bits are used for CSI feedback, which can achieve more performance gains as analyzed above. With the further increase of sensing SNR, the allocation of feedback resource is unchanged, while more and more time is

TABLE III
JOINT OPTIMIZATION WITH DIFFERENT SENSING SNRS

	ξ	-6dB	-4dB	-2dB	0dB
Traditional Algorithm	B	3	3	3	3
	L	3	3	3	3
	τ	5.0	5.0	5.0	5.0
Algorithm 1	B	2	3	3	3
	L	7	3	3	3
	τ	3.3875	2.4268	1.0250	0.5883
Algorithm 2	B	2	6	6	6
	L	4	2	2	2
	τ	4.9347	3.3804	1.4812	0.8361

used for spectrum accessing. Examining the resultant average sum rate in Fig.4, it is found that algorithm 2 performs even worse than traditional algorithm when ξ is equal to -6dB, this is because feedback resource is under-utilized based on the allocation scheme in this case. Specifically, the total number of used feedback bits is 12, but there are 3 bits left that are not enough to enlarge codebook size and are not allowed to be used for sensing because the upper of the number of sensing bits is approached. As sensing SNR increases, algorithm 2 outperforms the other two algorithms quickly. More interestingly, algorithm 2 achieves the performance advantage even with small number of feedback bits, which shows its high feedback utilization efficiency.

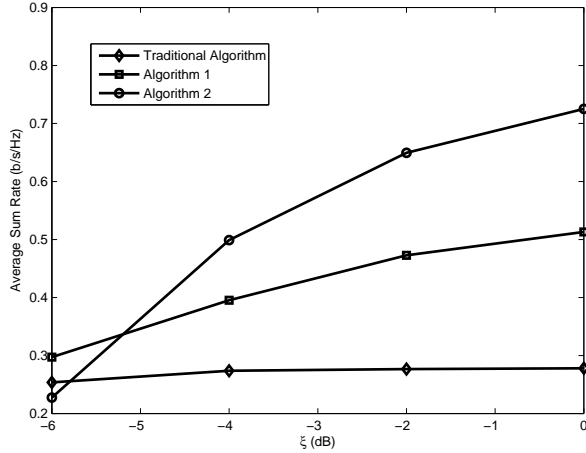


Fig. 4. Comparison of average sum rate with different sensing SNRs.

V. CONCLUSIONS

A major contribution of this paper is the construction of a joint optimization framework of spectrum sensing and accessing in a multiuser MISO cognitive network. Based on this framework, we present two algorithms to jointly allocate time and feedback resources, namely determining the duration of sensing time, the number of sensing users, the number of accessing users, and CSI quantization codebook size, by maximizing the average sum transmission rate. When given a performance requirement, this framework reveals the intrinsic relationship of these two resources. As a result, the resource transformation can be realized according to the characteristics of the considered network.

APPENDIX

For limited feedback ZFBF, since cognitive BS only has partial CSI of the accessing users, there is still residual interuser interference. In order to achieve the pdf of SINR, it is necessary to reveal the relationship between codebook size and the residual interference. According to the theory of random vector quantization [33], the relationship between the original and the quantized channel direction vectors is given by

$$\tilde{\mathbf{h}}_k = \sqrt{1-a}\hat{\mathbf{h}}_k + \sqrt{a}\mathbf{s}, \quad (28)$$

where $\hat{\mathbf{h}}_k$ is the optimal quantization codeword based on the codeword selection criteria (11). $a = \sin^2(\angle(\tilde{\mathbf{h}}_k, \hat{\mathbf{h}}_k))$ is the magnitude of the quantization error, and \mathbf{s} is a unit norm vector isotropically distributed in the nullspace of $\hat{\mathbf{h}}_k$, and is independent of a . Therefore, the interference term from the u th user to the k th user can be expressed as

$$\begin{aligned} |\mathbf{h}_k^H \mathbf{w}_u|^2 &= \|\mathbf{h}_k\|^2 |\tilde{\mathbf{h}}_k^H \mathbf{w}_u|^2 \\ &= \|\mathbf{h}_k\|^2 \left((1-a) |\hat{\mathbf{h}}_k^H \mathbf{w}_u|^2 + a |\mathbf{s}^H \mathbf{w}_u|^2 \right. \\ &\quad \left. + 2\sqrt{a(1-a)} |\mathbf{w}_u^H \hat{\mathbf{h}}_k \mathbf{s}| \right) \\ &= a \|\mathbf{h}_k\|^2 |\mathbf{s}^H \mathbf{w}_u|^2, \end{aligned} \quad (29)$$

where (29) follows from the fact that both \mathbf{w}_u and \mathbf{s} are in the nullspace of $\hat{\mathbf{h}}_k$, namely $\hat{\mathbf{h}}_k^H \mathbf{w}_u = 0$ and $\mathbf{s}^H \hat{\mathbf{h}}_k = 0$. Substituting (29) into (13), we have

$$\begin{aligned} \rho_k &= \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{1/\gamma + a \|\mathbf{h}_k\|^2 \sum_{u=1, u \neq k}^{N_t} |\mathbf{s}^H \mathbf{w}_u|^2} \\ &\stackrel{d}{=} \frac{\chi_2^2}{1/\gamma + a \|\mathbf{h}_k\|^2 \sum_{u=1, u \neq k}^{N_t} \beta(1, N_t - 2)} \end{aligned} \quad (30)$$

$$\stackrel{d}{=} \frac{\chi_2^2}{1/\gamma + \sum_{u=1, u \neq k}^{N_t} \Gamma(N_t - 1, \delta) \beta(1, N_t - 2)} \quad (31)$$

$$\stackrel{d}{=} \frac{\chi_2^2}{1/\gamma + \delta \chi_{2(N_t-1)}^2}, \quad (32)$$

where $\delta = 2^{-\frac{B}{N_t-1}}$ and $\stackrel{d}{=}$ denotes the equality in distribution. $\beta(x, y)$ represents the Beta distribution, whose probability density function is given by $g(t) = \frac{t^{x-1}(1-t)^{y-1}}{B(x, y)}$, where $B(x, y) = \frac{(x-1)!(y-1)!}{(x+y-1)!}$ is the Beta function. (30) follows from the facts that \mathbf{w}_k of unit norm is independent of $|\mathbf{h}_k^H \mathbf{w}_k|^2$, so $\mathbf{h}_k^H \mathbf{w}_k$ is a complex Gaussian distributed random variable with zero mean and unit variance. Then $|\mathbf{h}_k^H \mathbf{w}_k|^2$ is χ_2^2 distributed. In addition, $|\mathbf{s}^H \mathbf{w}_u|^2$ is $\beta(1, N_t - 2)$, because \mathbf{s} and \mathbf{w}_u are i.i.d. isotropic vectors in the $N_t - 1$ dimensional null space of $\hat{\mathbf{h}}_k$ [33]. (31) is derived since $a \|\mathbf{h}_k\|^2$ is $\Gamma(N_t - 1, \delta)$ distributed according to the theory of quantization cell approximation [34]. Moreover, (32) holds true since the product of a $\Gamma(N_t - 1, \delta)$ distributed random variable and a $\beta(1, N_t - 2)$ distributed random variable is $\delta \chi_{2(N_t-1)}^2$ distributed [35]. Note that the sum of $N_t - 1$ independent χ_2^2 distributed random variables

is $\chi_{2(N_t-1)}^2$ distributed. Let $y \sim \chi_{2(N_t-1)}^2$ and $z \sim \chi_2^2$, we can derive the cdf and pdf of ρ_k as follows

$$\begin{aligned}
 F_{\rho_k}(x) &= P\left(\frac{z}{1/\gamma + \delta y} \leq x\right) \\
 &= \int_0^\infty F_{Z|Y}(x(1/\gamma + \delta y)) f_Y(y) dy \\
 &= \int_0^\infty (1 - \exp(-x(1/\gamma + \delta y))) \\
 &\quad \times \frac{y^{N_t-2}}{\Gamma(N_t-1)} \exp(-y) dy \\
 &= 1 - \frac{\exp(-x/\gamma)}{(1 + \delta x)^{N_t-1}}, \quad (33)
 \end{aligned}$$

and

$$\begin{aligned}
 f_{\rho_k}(x) &= F'_{\rho_k}(x) \\
 &= 1/\gamma \exp(-x/\gamma) (1 + \delta x)^{-(N_t-1)} \\
 &\quad + \delta(N_t-1) \exp(-x/\gamma) (1 + \delta x)^{-N_t}, \quad (34)
 \end{aligned}$$

respectively, where $F_{Z|Y}(\cdot)$ is the conditional cdf of z when given y , $f_Y(\cdot)$ is the pdf of y , $\Gamma(\cdot)$ is the Gamma function. (33) holds true because $\frac{\exp(-(1+\delta x)y)((1+\delta x)y)^{N_t-2}}{\Gamma(N_t-1)}$ is the pdf of $(1 + \delta x)y$.

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